PRACTICE SHEET NO. 1 (Based on Chapter 1)

Calculate the location of center of gravity of the sections shown in the following figures:

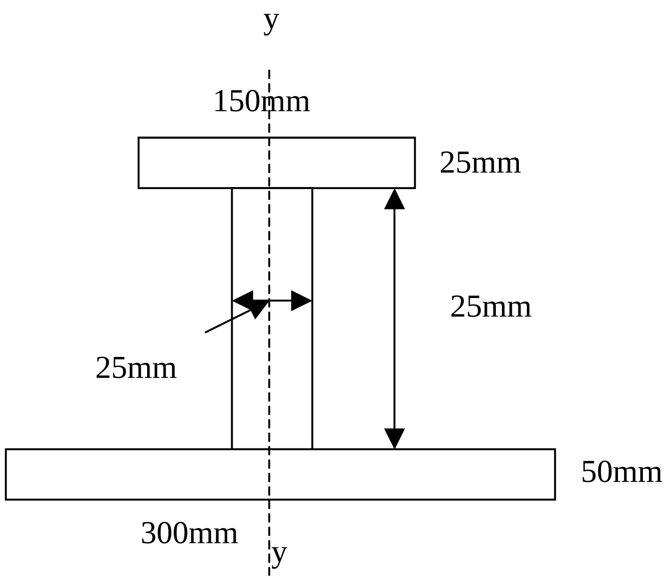


FIG. 1

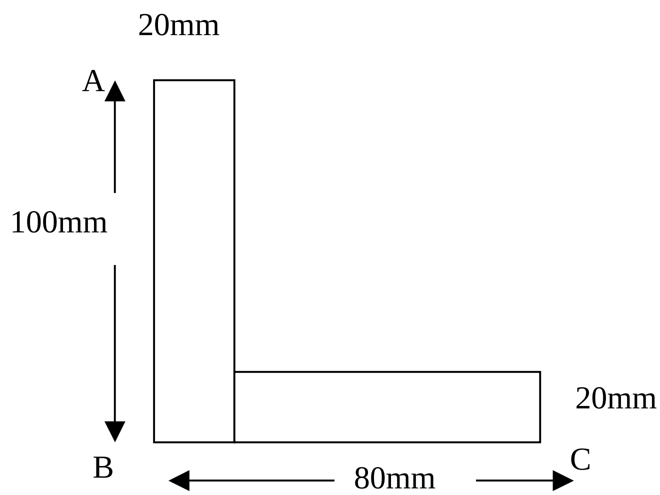


FIG. 2

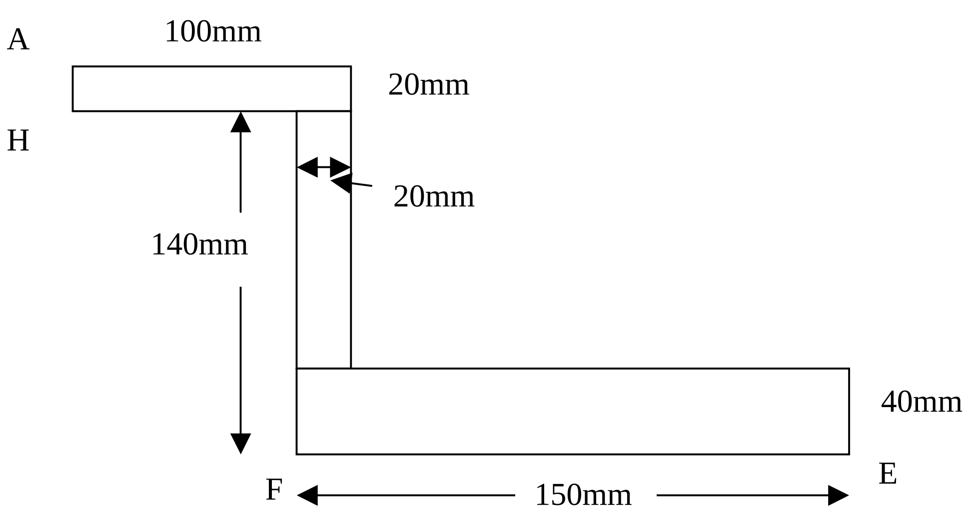


FIG. 3

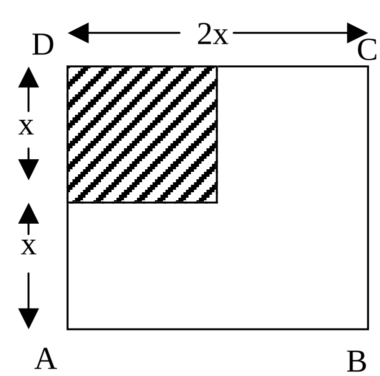
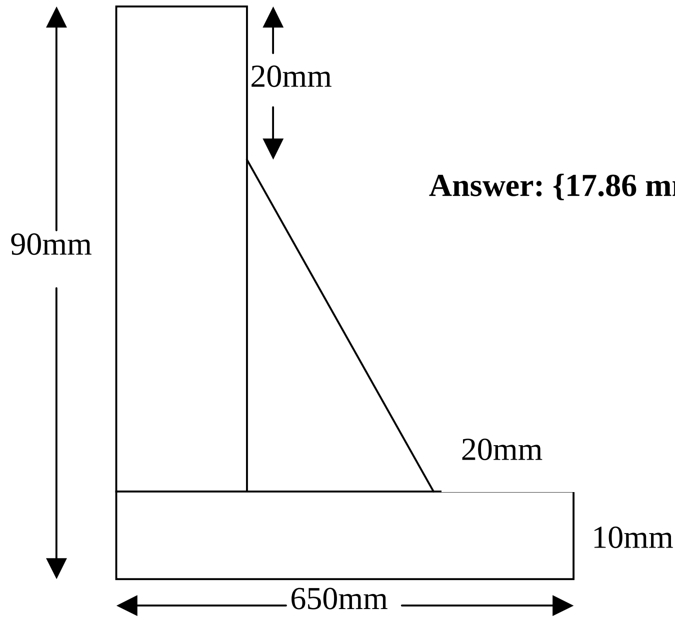


FIG. 4

ANSWERS: (1) 88.8 mm from bottom face of the flange; (2) 25 mm from face AB, 35 mm from face BC; (3) 121 mm from face AH,60 mm from face FE; (4) 5/12 times the square ABCD



Answer: {17.86 mm from face AB, 36.07 mm from face BC}

FIG. 5

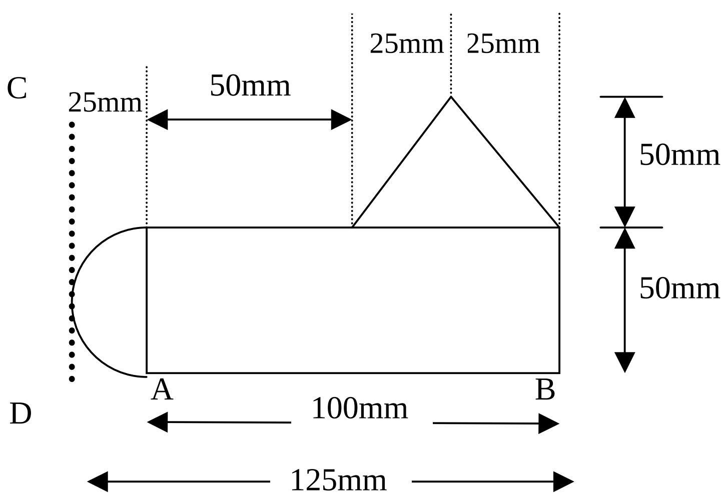
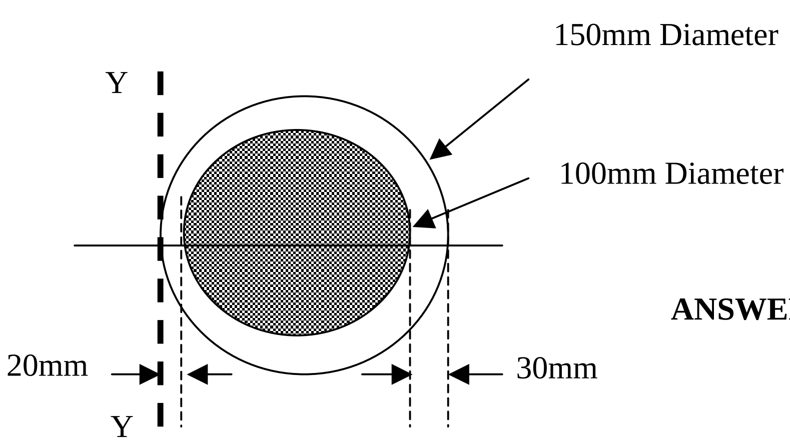


FIG. 6

ANSWER: (71.1 mm from face CD, 32.2 mm from face AB)



ANSWER: (79 mm from YY)

FIG. 7

PRACTICE SHEET NO. 2 (Based on Chapter 1)

CALCULATE THE SECOND MOMENT OF AREA OF THE FOLLOWING SECTION ABOUT THEIR CENTROIDAL AXIS XX:

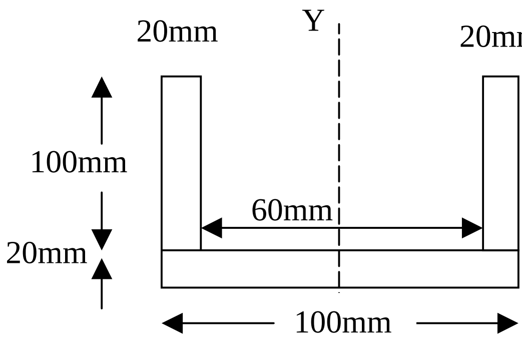


FIG. 1

($I_{XX} = 8.2 \times 10^6$ mm^4)

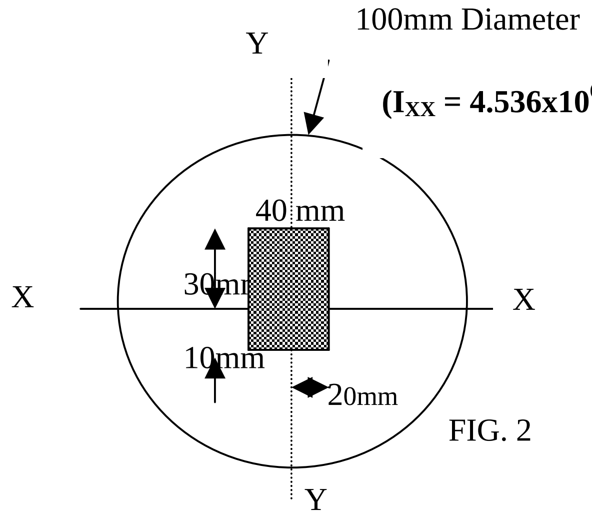


FIG. 2

($I_{XX} = 4.536 \times 10^6$ mm^4)

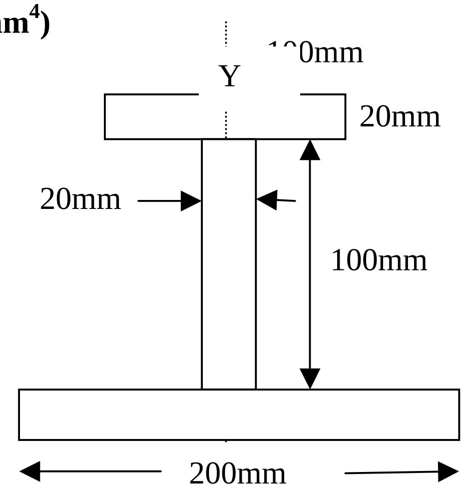


FIG. 3

($I_{XX} = 821.667 \times 10^6$ mm^4)

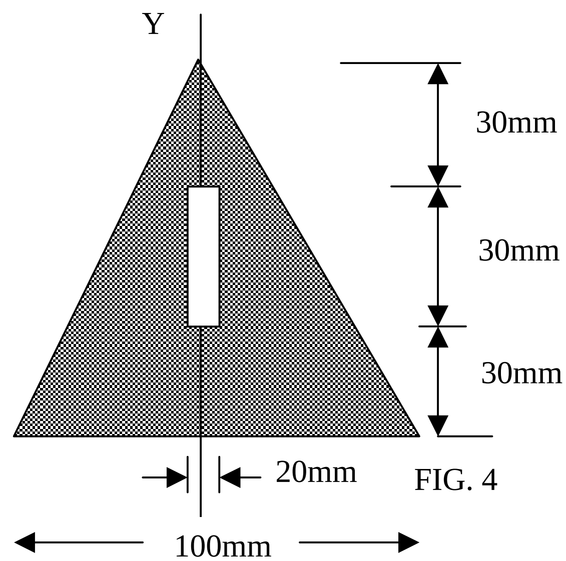


FIG. 4

($I_{XX} = 1.824 \times 10^6$ mm^4)

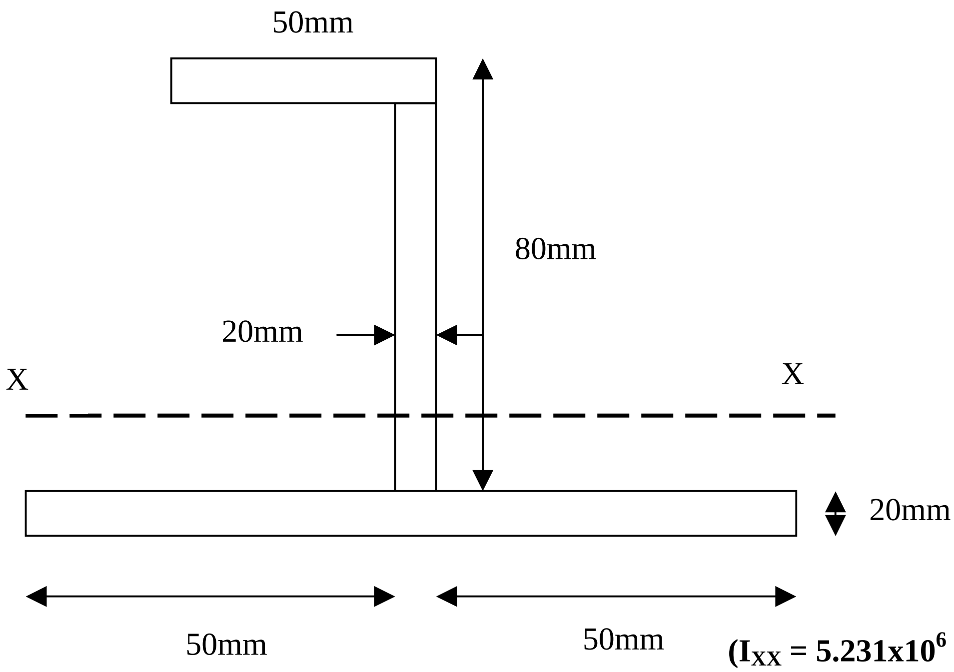
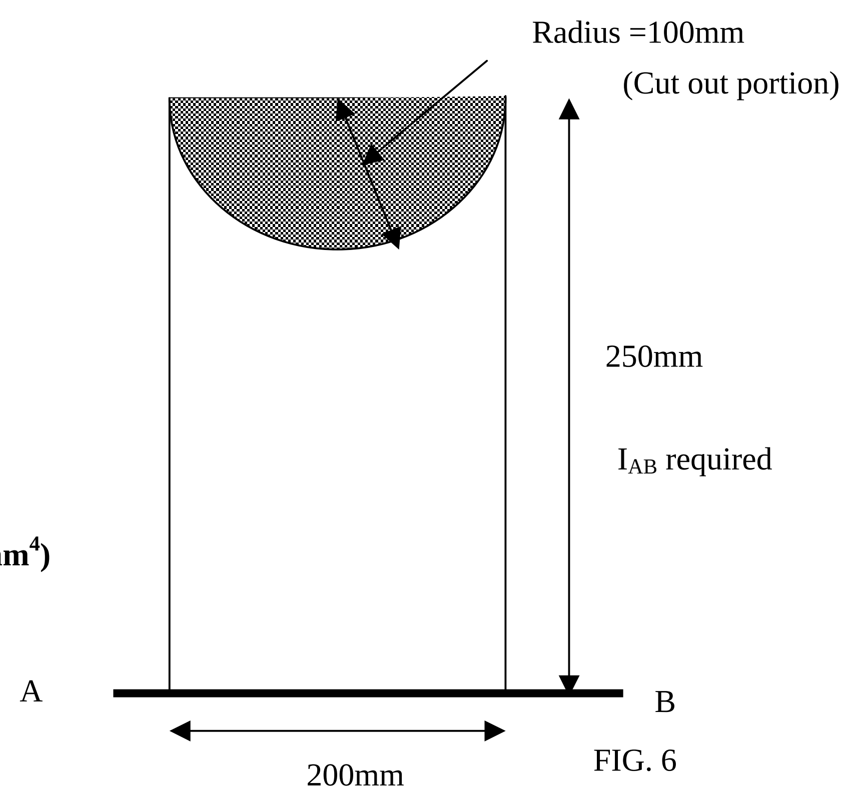


FIG. 5

(I_{XX} = 5.231x10^6 mm^4)



(I_{AB} = 354x10^6 mm^4)

FIG. 6

PRACTICE SHEET NO. 3 (Based on Chapters 4, 6, 7)

1. A steel plate, 200 mm deep by 20 mm wide, is fixed firmly to, and symmetrically between two timber beams, each 250 mm deep by 100 mm wide, to form a composite beam. The elastic moduli of the steel and timber are 200×10^3 and 12.5×10^3 N/mm^2 respectively and the allowable stresses in the steel and timber are 140 and 8.4 N/mm^2 respectively.

If the beam is simply supported over a span of 4 m, find (a) the intensity of the safe uniformly distributed load and (b) the value of two vertical equal concentrated loads which are each at a distance of 1.2 m from the ends. Compare the values of the total loads in (a) and (b).

Find the depth of steel plate required if the uniformly distributed load is increased by 15 %.

{15.918 kN/m, 26.53 kN, 20 % ; 220.122 mm}

2. Two steel plates, each 100 mm wide by 10 mm thick, are welded together to form a symmetrical tee-section which acts as a vertical column. The column carries a vertical concentrated load at the centroid of its web. If compressive stresses are not to exceed
100 N/mm^2, find the maximum value of the load. Find also the minimum value of stress on the cross section and illustrate the stress distribution on a diagram.

If the column is 2 m high determine the value of the horizontal force that would be needed at the top to ensure uniformity of stress throughout the cross section. Had this force been the only one on the column , what would have been the maximum tensile and compressive stresses? Illustrate these on a stress distribution diagram.

{71.16 kN, -8.56 N/mm^2, 0.97845 kN, 64.426 and -27.017 N/mm^2}

3. A uniform horizontal beam, 16 m long and carrying a uniformly distributed load of 20 kN/m, is freely supported at two points 10 m apart. If the maximum bending moment on the beam is to be as small asa possible, find the positions of the

supports. Hence, determine the values of the maximum and other principal bending moments. Find also the positions of the maximum bending moment and points of contraflexure and sketch the bending moment diagram showing all the relevant values of bending moment and distance.

{3.93 and 2.07 m from ends; ±154.4 and -42.84 kN m;5.5578 and 13.418 m from Left Hand End}

4. Explain, using sketches, the term Equivalent Second Moment of Area and Equivalent Beam Section.

A horizontal beam, 48 mm wide and 30 mm deep, is made by fixing a rectangular steel section firmly on top of a brass one. If the moduli of elasticity are 207×10^3 and 92×10^3 N/mm^2 for steel and brass respectively, find the thickness of the steel and brass section such that the neutral plane of the beam is at their common surface.

Determine also the safe uniformly distributed load on a simply supported span of 0.9 m if the allowable stresses are 105 and 69 N/mm^2 for steel and brass respectively. Sketch the stress distribution across the section of the composite beam.

{12 and 18 mm, 5.005 kN/m}

5. A beam section has a top flange 100 mm wide by 25 mm thick, a bottom flange 300 mm wide by 25 mm thick and a web 200 mm deep by 15 mm thick. If the beam is simply supported on a horizontal span of 5 m and the allowable stresses are 46 N/mm^2 and 23 N/mm^2 in compression and tension respectively, find the safe uniformly distributed load.

{9.863 kN/m}

6. A uniform horizontal cantilever beam, whose span is five times its depth, has an isosceles triangular cross-section. The cross-section has a width 'b' at the top and tapers to a vertex at the bottom, the depth being equal to 'd'. Derive the second moment of area of the cross-section about its horizontal centroidal axis and determine the ratio of the maximum stresses.

For b = 240 mm, d = 450 mm and a maximum allowable stress of 10.5 N/mm^2, find (a) the safe uniformly distributed load and (b) the safe value of a vertical concentrated load 'W' if one such load is located at mid-span and another at the free end.

{$(bd)^3/36$; 1/2; 8.4 kN/m; 6.3 kN}

7. A uniform horizontal beam ABC, 13 m long, is supported at one end A and at a point B which is 10 m from A. It carries a uniformly distributed load of 40 kN/m from a point 6 m from A to one 11 m from A, a vertical concentrated load of 80 kN at a point 2 m from A and another one of 20 kN at C.

Calculate the significant values of shearing force, illustrate these clearly on a diagram and indicate the point or points at which the maximum bending moment will occur. Using the shearing force diagram, calculate the significant values of bending moment and show these on a diagram. Find also the position of zero bending moment in the span.

{S_{AB} = 88; S_{BA} = 152 and S_{BC} = 60 kN; M_{max} = 208.8 kN m at 6.2 m from A; M_B = -80 kN m; Point of Contraflexure is 0.57 m to left of B}

8. A uniform horizontal beam, 13 m long, is supported at B and C which are 2 and 10 m respectively from A. It carries a uniformly distributed vertical load of 20 kN/m from A to C and a concentrated vertical load of 24 kN at D. Calculate the significant values of shearing force and bending moment and points of contraflexure. Sketch the appropriate diagrams to scale.

{S_{BA} = -40; S_{BC} = 76; S_{CB} = -84 and S_{CD} = 24 kN; M_B = -40 and M_C = -72 kN m; M_{max} = 104.4 kN m at 3.8 m from B; 0.569 and 7.031 m to the right of B}

9. A uniform horizontal beam ABC, 8 m long, is fixed at A, pinned at the centre B and supported on rollers at C. It carries a vertical concentrated load of 20 kN at the centre of AB and a uniformly distributed load of 30 kN/m from the centre of BC to the support C.

Calculate the positions and magnitudes of the maximum sagging and hogging bending moments and sketch the dimension shearing force and bending moment diagrams.

{67.5 kN m at 1.5 m from C and -100 kN m at A}

10. A uniform horizontal beam, 8 m long, carries a uniformly distributed load of 37.5 kN/m. It is simply supported at A and a point B from the other end such that its mid-point ai a point of contraflexure. Find the distance BC and determine the principal values of shearing force and bending moment.

{2.67 m; S_{AB} = 75, S_{BA} = -125 and S_{BC} = 100 kN; M_{ab} = 75 and M_B = -133.37 kN m}

11. A uniform horizontal cantilever, 3 m long, is 60 mm wide throughout its length, the depth varying uniformly from 180 mm at the fixed end to 60 mm at the free end. If the vertical concentrated load of 4 kN is carried at the free end, find the position of the most highly stressed cross section and thus deduce the maximum bending stress.

{mid span; 41.67 N/mm^2}

12. A uniform horizontal beam ABCD, simply supported at B and C, carries vertical loads of 50 and 30 kN at A and D respectively and a uniformly distributed load of 20 kN/m throughout its length. If the beam is 12 m long and BC = 8 m, find the positions of the supports such that their reactions are equal. Hence calculate the principal values of shearing force and bending moment and find the positions of the points of contraflexure.

{AB = 1.625 and CD = 2.375; S_{AB} = -82.5, S_{BC} = 77.5, S_{CB} = -82.5,
and S_{CD} = 77.5 kN; M_B = -107.565, M_{bc} = 42.5 and M_C = -127.656 kNm
1.813 and 5.937 m to right of B}

PRACTICE SHEET NO. 4 (Based on Chapter 3)

1.Using graphical method, find the values of the forces in the statically determinate frame shown in Figure 1.

(B1 = B6 = +3.75, B3 = B4 = +3.00, B8 = +7.5, A1 = D2 = -2.25, D5 = -3.75, D7 = D8 = -4.5, I2 = -2.0, 23 = -1.25, 34 = 0, 45 = +1.25, 56 = -1.0, 67 = +1.25, and 78 = -7.0 kN)

2. Find graphically the forces in the members of the cantilever frame shown in Figure 2 and the magnitude and directions of the reactions at the supports.

(A2 = A3 = +121.24, A6 = A7 = +17.32, EI = -200.0,
D4 = C5 = -69.28, B7 = -20.0, I2 = 23 = 0.0, 34 = -120.0 , 45 = +30.0, 56 = +60.0 and 67 = 0 kN)
(EF = 200 kN at 60^0 to the vertical,FA=173.2 horizontal)

3.Figure 3 shows a frame simply supported at H and E and vertically loaded at A,G and F. Determine the reactions at the supports and, using the method of joint equilibrium, find the forces in the members. Tabulate these forces and show them clearly on a sketch of a frame.

(45 and 30 kN; AB = 17, BC = -32, CD = -40, DE = -50, EF = 40,
FG = 32, GH = -8, HA = -8, BH = -45, CG = -6, DF = 30, BG = 50 and CF = 10 kN)

4. Figure 4 shows a plane frame simply supported at A and D and vertically loaded at B and C. Using method of Tension Coefficients, calculate the forces in the members of the frame and show them on a clear sketch of the frame. All calculations must be clearly shown.

(AB = -120, BC = -96, CD = -105, DE = EF = 84, FA = 96, BF = -9, CE = 0, CF = 15)

5. Figure 5 shows a plane cantilever frame hinged to a vertical wall at A and J. Using the method of joint equilibrium find the forces in the members of the frame and the reactions at the supports. The stages in the calculations must be clearly shown and the forces and reactions illustrated on a sketch of the frame.

(AB = BC = 144, CD = 64, DE = 16, EF = -20, FG = -16, GH = -64, HJ = -160, AH = -20, BH = -24, CH = -100, CG = 36, DG = -60, DF = 12; 128.56 at $5^0 21'$ Horizontal and 160)

6. Figure 6 shows a vertical frame pinned at the ground at A and H. Use the method of joint equilibrium to find the forces in the members of the frame due to the loading and, hence, deduce the reactions at A and H. All calculations must be clearly shown and the forces and reactions must be illustrated on a neat sketch of the frame.

(AB = 86, BC = 14, CD = DE = -10, EF = -34, FG = -106, GH = -226,
CF = -30, AG = 130, BF = 78, CE = 26; 211.98 at $13^0 38'$ to vertical and 226)

7.Figure 7 shows a frame hinged to a vertical wall at A and G and vertically loaded at B,C and D. Use the method of joint equilibrium to determine the forces in the members of the frame, showing clearly all calculations. Hence, deduce the magnitude and direction of the reactions at the supports and show all the forces and reactions on a sketch of the frame.

(AB = BC = 48, CD = 12, DE = -13, EF = -12, FG = -48.214, AF = -10.214, BF = -10,
CE = -5, CE = -39)

8. The plane, pin-jointed frame shown in Figure 8 is hinged to a rigid ceiling at D and E. Use the method of joint equilibrium to determine the forces in the members of the frame and hence deduce the reactions at D and E. All steps in the calculations must be clearly shown and the forces illustrated on a sketch of the frame.

(AB = 10, BC = 8, CD = 32, EF = -72, FG = -32, GA = -8, BG = -6, CF = -18, CG = 30,
DF = 50; 78 AT $22^0 37'$ to Vertical and 72)

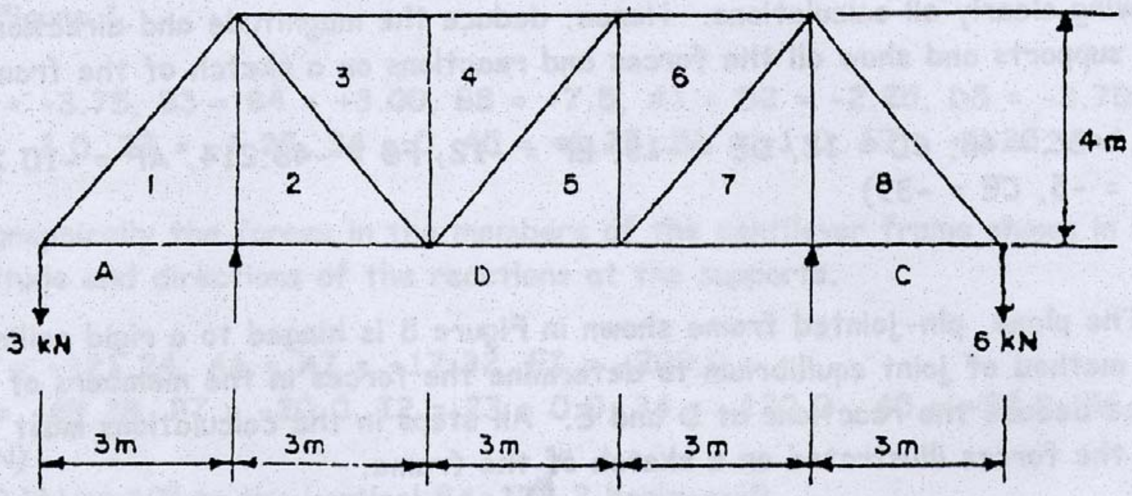
9. Determine the magnitude and direction of the reactions at A and D of the loaded plane frame shown in Figure 9 and then use the method of tension coefficients to find the forces in the members. Show the reactions and forces on a neat sketch of the frame.

(4 and 8.944 at $26^{0}34'$ to Vertical; AB = 14.422,BC = 6.667, CD = 14.422, DE = -5.657,
EA = -16.971, BE = CE = 9.615)

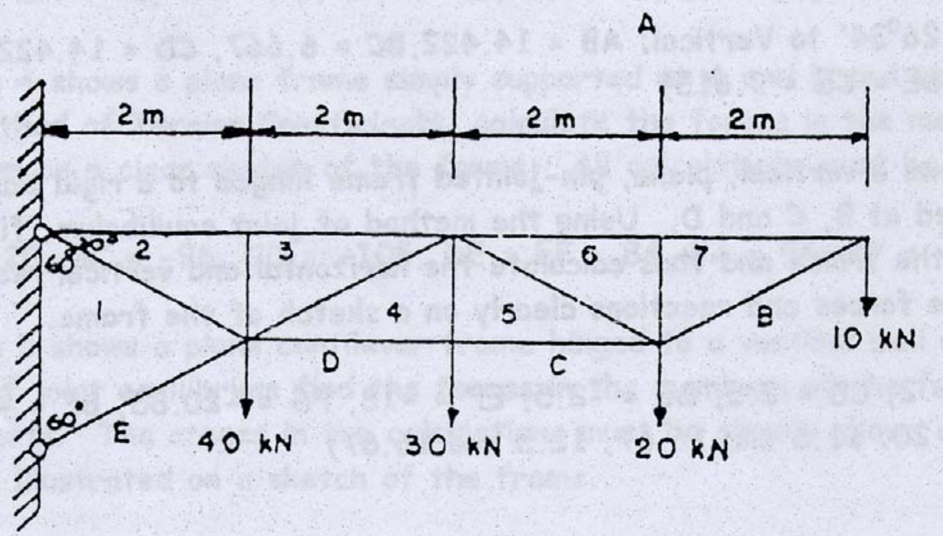
10.Figure 10 shows a vertical, plane, pin-jointed frame hinged to a rigid base at A and C and horizontally loaded at B, C and D. Using the method of joint equilibrium, find the forces in the members of the frame and thus calculate the horizontal and vertical reactions at the hinges. Show the forces and reactions clearly on a sketch of the frame.

(AB = 22.5, BC = 2, CD = 2.5, DE = -2.5, EF = -18, FG = -20.83, BF = -10.5, CE = -10.5, AF = -2.4, BE = 20; 11.5 and 16.67, 12.5 and 16.67)

11. Figure 11 shows a plan of a three-dimensional frame which is pinned to level ground at A,B,C and D the horizontal member EF being 4 m above the ground. The frame carries a vertical load of 12 kN at E and a horizontal load of 8 kN at F.
Find the forces in the members of the frame and the vertical reactions at the ground level.
(AE = -9, BE = -9, CF = -6, DF = 0, EF = -6, AF = 6.928; A = 2, B = 6, c = 4 and D = 0.)


3
4
6
4 m
1
2
5
7
8
A
D
C
3 kN
6 kN
3m
3m
3m
3m
3m

FIG·1


A
2 m
2 m
2 m
2 m
30°
60°
2
3
6
7
1
4
5
B
10 kN
D
C
60°
E
40 kN
30 kN
20 kN

FIG· 2

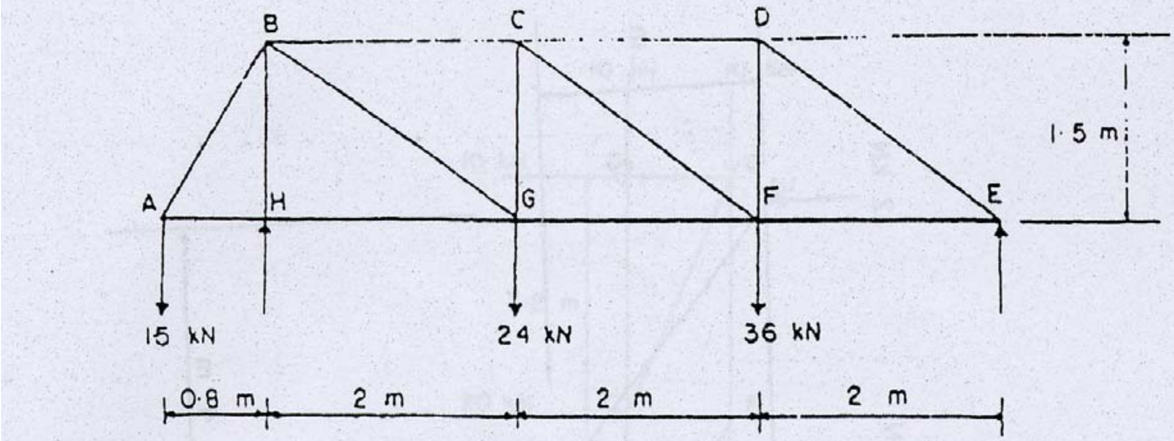

B
C
D
1·5 m
A
H
G
F
E
15 kN
24 kN
36 kN
0·8 m
2 m
2 m
2 m

FIG. 3

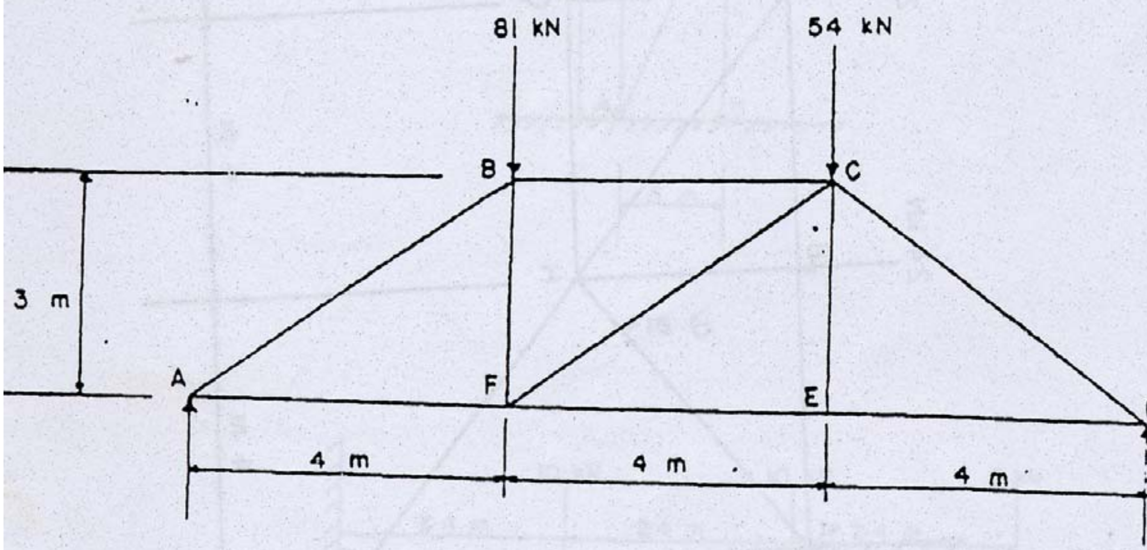

81 kN
54 kN
B
C
3 m
A
F
E
D
4 m
4 m
4 m

FIG. 4

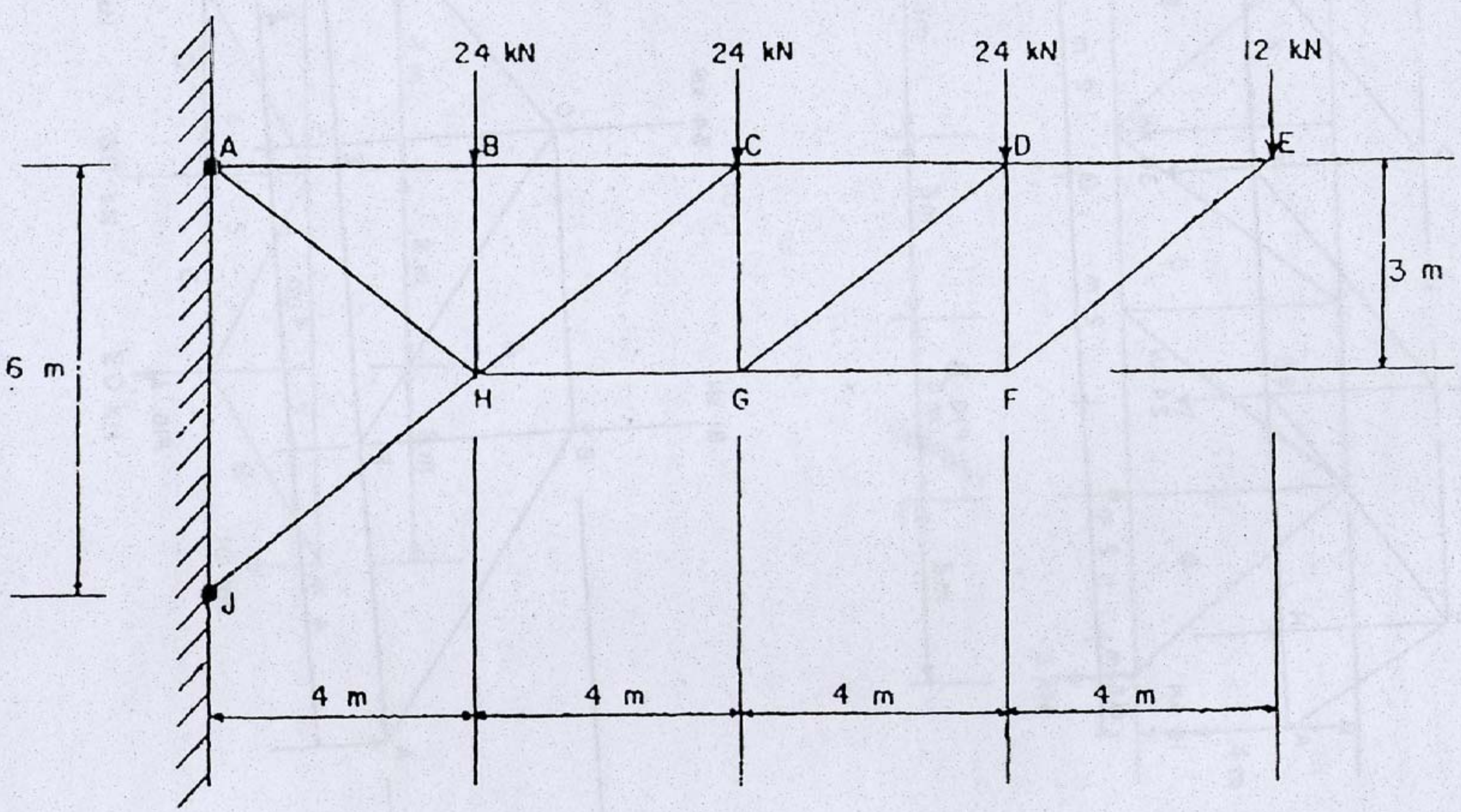
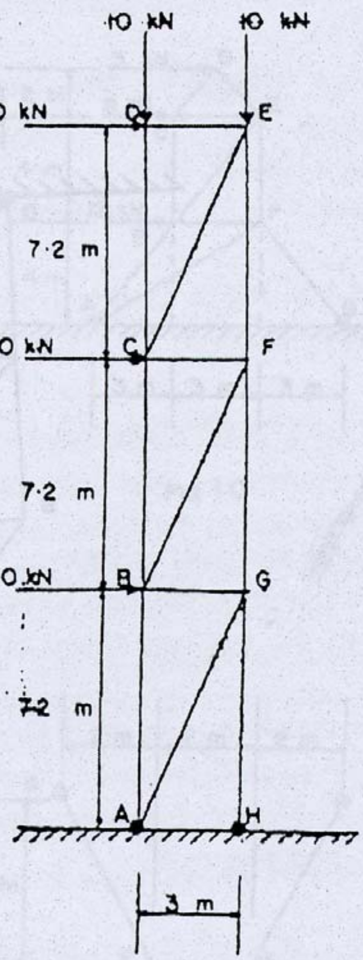
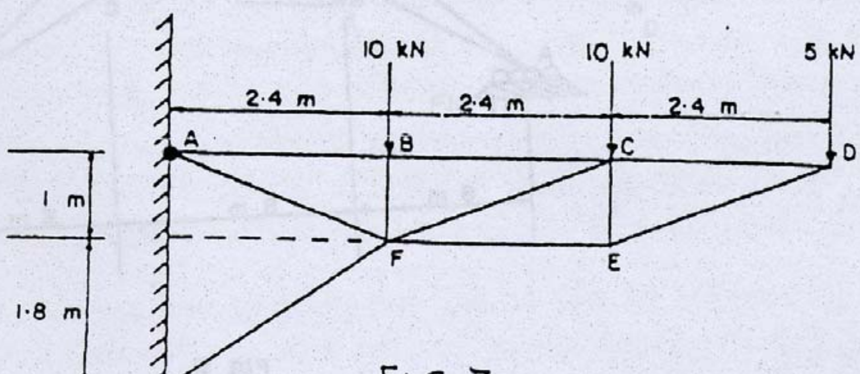


FIG. 5



10 kN
10 kN
10 kN
D
E
7·2 m
20 kN
C
F
7·2 m
20 kN
B
G
7·2 m
A
H
3 m

FIG. 6



10 kN
10 kN
5 kN
2·4 m
2·4 m
2·4 m
A
B
C
D
1 m
F
E
1·8 m

FIG. 7

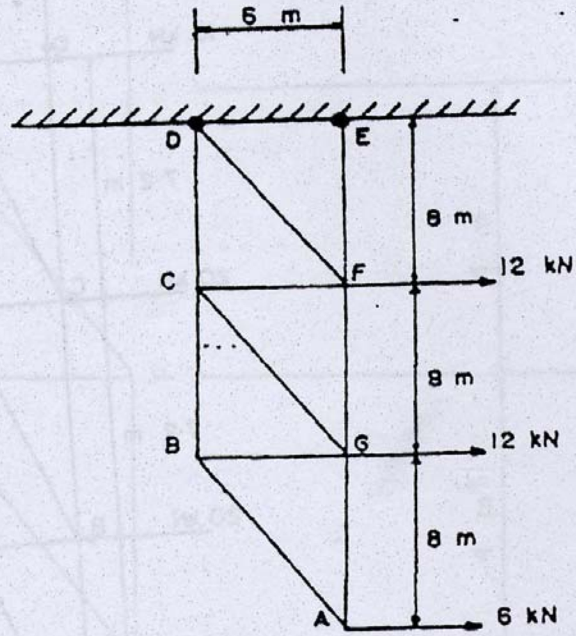


FIG 8

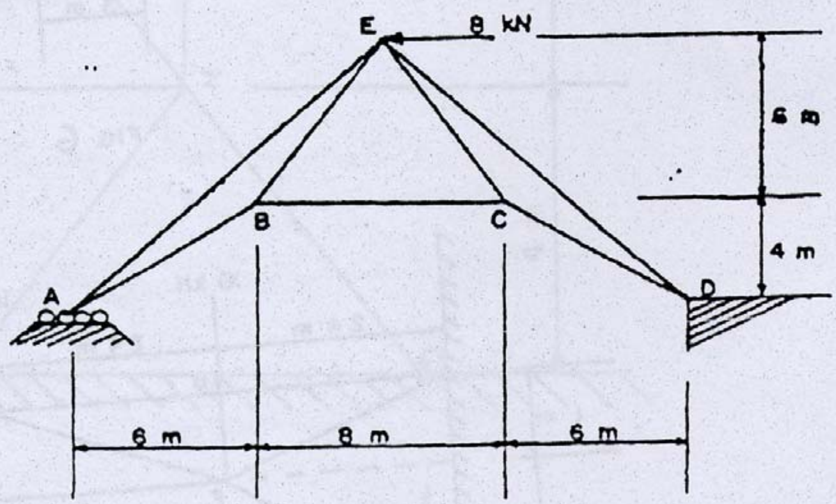


FIG 9

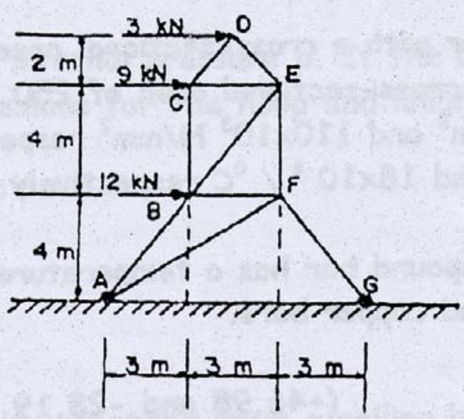


FIG 10

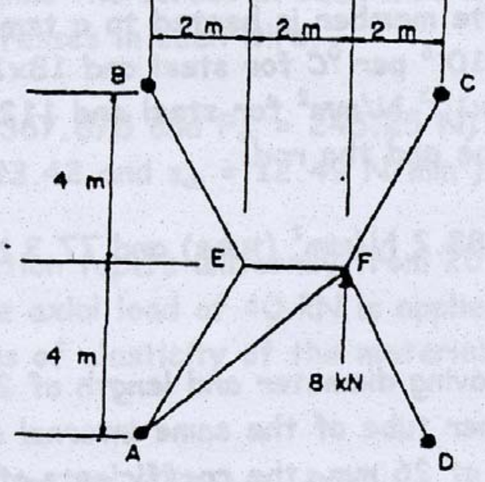


FIG 11

PRACTICE SHEET NO. 5 (Based on Chapter 5)

1. A steel bar with a cross-sectional area of 300 m^2 is soldered between two copper ones each having a cross-sectional area of 250 mm^2. The steel and copper have elastic moduli of 210×10^3 N/mm^2 and 110×10^3 N/mm^2 respectively and coefficients of linear expansion of 12×10^{-6} $/^0C$ and 18×10^{-6} $/ ^0C$ respectively.

If the compound bar has a temperature increase of 80^0 C, calculate the thermal stresses in the steel and copper bars.

(+46.98 and -28.19 N/mm^2)

2. A hollow steel tube encloses a copper rod of the same length and is rigidly joined to it at each end. The tube has internal and external diameters of 20 and 25 mm respectively, the rod has a diameter of 16 mm and there are no longitudinal stresses at a temperature of 15^0C.

If the composite member is heated to a temperature of 200^0C, the coefficients of linear expansion are 12×10^{-6} per ^0C for steel and 18×10^{-6} per ^0C for copper and the moduli of elasticity are 210×10^3 N/mm^2 for steel and 112×10^3 N/mm^2 for copper, calculate the stresses in the tube and the rod.

[88.2 N/mm^2 (tube) and 77.3 N/mm^2 (rod)]

3. A steel rod, having diameter and length of 20 mm and 0.4 m respectively, is securely fixed inside a copper tube of the same internal diameter and length. The tube has an external diameter of 26 mm, the coefficients of linear expansion are as in Problem No.2 above and the moduli of elasticity are 200×10^3 N/mm^2 for steel and 110×10^3 N/mm^2 for copper.

Find the stresses in the tube and rod and the change in length of the compound bar for (a) an axial compressive force of 40 kN and (b) an increase in temperature of 80^0C.

(50.763 and 92.297 N/mm^2 for the tube and the rod; 0.185 mm)
(-38.275 and 26.41 N/mm^2 for the tube and the rod; 0.437 mm)

4. A mass of 150 kg is suspended by three vertical wires, each having a cross-sectional area of 8 mm^2 and carrying an equal share of load. The central wire is of aluminum and the other two of steel, the coefficients of linear expansion and the elastic moduli being $24x10^{-6}$ per ^0C and $70x10^3$ N/mm^2 for aluminum and $12x10^{-6}$ per ^0C and $210x10^3$ N/mm^2 for steel.

If the temperature is raised by 50^0C, calculate the stresses in the wires. Find also the
rise in temperature that would just cause the aluminum wire to go slack.

(25.3 and 79.3 N/mm^2 for aluminum and steel; 87.67^0C)

5. A cylindrical boiler is subjected to an internal pressure p. If the boiler has a mean radius r and a wall thickness t, derive expressions for the hoop and longitudinal stresses in the wall.

If the Poisson's ratio for the material is 0.3, find the ratio of the hoop strain to the longitudinal strain and compare it with the ratio of stresses.

(Strain ratio = 4.25; Stress ratio = 2.00)

6. A rigid body having a mass of 100 kg is suspended from a ceiling by three vertical wires in one plane such that one wire is centrally located and the others are symmetrically located on each side of the centre. The central wire is made of copper while the other two are made of steel, the latter having a cross-sectional area each equal to four fifths that of the former.

If the body remains horizontal and the elastic moduli of copper and steel are $112x10^3$ and $210x10^3$ N/mm^2 respectively, determine the forces in each wire if the copper wire has a diameter of 5 mm, determine the stresses in each wire.

(F_{st} =367.875 and F_{cu} = 245.25 N)
(s_{st} = 23.42 and s_{cu} = 12.49 N/mm^2)

7. A specimen of circular cross-section tapers uniformly from 20 mm to 16 mm diameter over a length of 200 mm. When a tensile axial load of 40 kN is applied, the specimen lengthens an amount of 0.4 mm. Find the modulus of elasticity of the material.

(79.6×10^3 N/mm^2)

8. A steel ball of radius r is so machined in a lathe that it has equal and parallel flat surfaces on two opposite sides, the distance between these surfaces being 1.6 r. If an axial load W is applied to these faces, find the decrease in the distance between them.

{2.197 (W/prE)}

9. A cylindrical bar, 0.6 m long, is made up of a steel rod of 30 mm diameter concentrically attached at one end to a copper rod such that the copper rod is twice the length of the steel rod. Under an axial tensile force of 20 kN the extensions of each rod are equal. If the elastic moduli are 200×10^3 and 110×10^3 for steel and copper respectively, find the stresses in each material and the diameter of the copper rod.

(s_{st} = 28.3 and s_{cu} = 7.8 N/mm^2 ; d = 57 mm)

10. A rigid horizontal bar of negligible weight is 0.8 m long and suspended in position by vertical rods of equal length attached to its ends, one rod being of steel and the other of copper. The diameters and elastic moduli of the steel and copper rods are 12 mm and 210×10^3 N/mm^2 and 15 mm and 100×10^3 N/mm^2 respectively. Find the position of a vertical load of 5.4 kN on the bar which keeps it horizontal.

(0.3413 m from steel rod)

11. A bar of circular cross-section tapers uniformly down from a diameter of 30 mm to one of 16 mm. If the bar has a length of 0.5 m which elongates to an amount of 0.8 mm under an axial tensile force of 48 kN, find the elastic moduli. Hence, determine the elongation of a uniform rod of the same length and volume subjected

to the same force. Calculate the ratio of the maximum stresses in the two bars and draw the stress diagrams, showing the principal values.

(79.587×10^3 N/mm^2, 0.726 mm, 2.066)

12. Explain the meaning of the term "linear coefficient of expansion" as applied to a uniform bar and show how a thermal stress can be created in such a bar. Hence, derive a general expression for the state of thermal force in a compound bar formed of two different materials rigidly fixed together.

A steel tube, with inner and outer diameters of 30 and 40 mm respectively, encloses a copper rod of the same length which has a diameter of 20 mm. The tube and rod are rigidly fixed together. If the linear coefficients of expansion of steel and copper are 12×10^{-6} per ^0C and 18×10^{-6} per ^0C respectively and the moduli of Elasticity are 210×10^3 and 98×10^3 N/mm^2 respectively, find the stresses and forces in each material for an increase in temperature of 190^0C.

(Steel = 50.4 N/mm^2 and 27.7 kN; Copper = -88.2 N/mm^2 and - 27.7 kN)

13. Describe, with the aid of sketches, the behaviour of a uniform steel bar tested to fracture in a tensile testing machine and, hence, define the term "Modulus of Elasticity".

A cylindrical steel rod (diameter = 25 mm) is 0.42 m long and enclosed by a tube of the same length, such that the rod and tube are rigidly fixed to each other at the ends. The inner and outer diameters of the tube are 25 and 30 mm respectively and the compound bar is subjected to an axial tensile force of 80 kN. The Moduli of Elasticity of rod and tube are 210×10^3 and 98×10^3 respectively.

Calculate the stresses in the rod and the tube and find the extension of the compound bar.

{135.21 and 63.1 N/mm^2; 0.27 mm}

14. Derive expressions for the hoop and longitudinal stresses in a thin walled, cylindrical pipe with closed flat ends and subjected to internal pressure. Hence,

find expressions for the hoop and longitudinal strains, the ratio of these strains and the change in volume.

Such a pipe has an internal diameter of 80 mm, a wall thickness of 2 mm and a length of 1.3 m and is subjected to an internal pressure of 3.2 N/mm^2. The Modulus of Elasticity and Poisson's Ratio for the material are 210×10^3 N/mm^2 and 0.28 respectively. Find the hoop and longitudinal stresses and strains, the ratio of the strains and the change in volume.

{65.6 and 32.8 N/mm^2; 268.65 and 68.72 μe;3.909,4060 mm^3}

15. A brass bar, having a diameter of 70 mm and a length of 0.2 m at 20 degree C, is placed between two rigid stops 0.2 m apart at that temperature. The elastic modulus and coefficient of linear expansion are 100×10^3 N/mm^2 and 18×10^{-6} per degree C respectively. If the temperature is increased to 132 degree C, what is the force induced in the bar? Determine also the temperature to which the bar may be heated if the maximum allowable compressive stress is 234 N/mm^2.

(-775.85 kN and 150^0C)

16. A steel rod, having a diameter of 20 mm, is placed vertically into a recess, 80 mm deep, such that it fits snugly. The rod is surrounded by a brass tube, having a length of 200 mm and inner and outer diameters of 35 and 40 mm respectively. The top of the brass tube is 0.07 mm above the top of the steel rod. The moduli of elasticity of brass and steel are 100×10^3 and 200×10^3 N/mm^2 respectively. If the allowable compressive stresses in brass and steel are 90 and 105 N/mm^2 respectively, determine the maximum value of an axial load applied through a rigid plate resting on the top of the tube. If the compressive stresses in the rod and tube are equal, how much will be the tube shortened ?

(51.19 kN and 0.23 mm)

17. A steel rod, 20 mm in diameter and 0.4 m long, is securely fixed within a copper tube of the same internal diameter and an external diameter of 26 mm. The tube and rod have the same length. The elastic modulus and coefficients of linear expansion of the tube are 110×10^3 N/mm^2 and 18×10^{-6}

per ^{0}C respectively while those of the rod are 200x10^3 N/mm^2 and 12x10^{-6} per ^{0}C respectively.

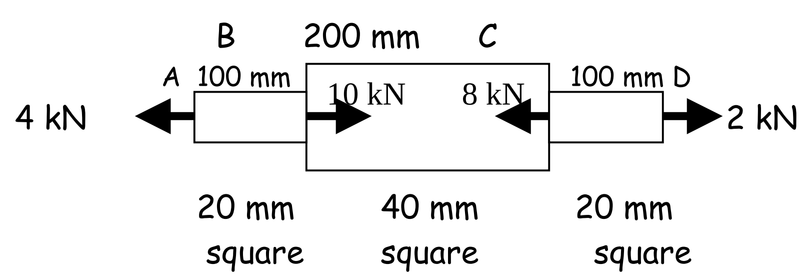
Find the stresses in the tube and rod and the change in length of the compound bar for (a) an axial compressive force of 40 kN and (b) an increase in temperature of 80^{0}C.

{(a) 92.297 (steel) and 50.76 (copper) N/mm^2; 0.185 mm; (b) 26.41 (steel) and
- 38.276 (copper) N/mm^2; 0.437 mm}

18. A mild steel rod was tested in a Universal Testing Machine. The diameter and the length of the test specimen was 10 mm and 200 mm respectively. It was found that when the rod was subjected to 10 kN load the rod's extension was 0.12 mm. The maximum load that could be applied to the specimen was 26 kN and the load beyond which stress-strain curve for the rod was not proportional was 11 kN. At fracture the 200 mm rod showed an extension of 61.5 mm and its diameter at this stage was measured as 5.7 mm. Calculate (a) the stress at limit of proportionality, (b) Young's Modulus, (c) percentage elongation and (d) percentage contraction of area of the rod at fracture.

{(a) 140.06 N/mm^2; (b) 212.2 X10^3 N/mm^2; (c) 30.75%; (d) 67.5%}

19. A steel rod ABCD of stepped sections is loaded as shown in the figure. The loads are all assumed to act along the centre line of the rod. Estimate the displacement of D relative to A. Assume E = 220 X10^3.



{ Total Elongation = 3.409×10^{-3} mm }

20. A member formed by connecting a steel bar (50 X50 mm; length = 300 mm) to an aluminium bar (100 X 100 mm; length 380 mm). Assuming that the bars are prevented from buckling sideways, calculate the magnitude of the compressive force P that will cause the total length of the member to decrease by 0.25 mm. Take E for steel and Aluminium as 210 kN/mm^2 and 70 kN/mm^2 respectively. What is the total work done by the force P.

{ P = 224.358 kN; Work done = 28.045 kN m }

21. Two vertical rods one of steel and the other of copper are each rigidly fixed at the top and are 500 mm apart. Diameters and lengths of each rod are 20 mm and 40 mm respectively. A cross bar fixed to the rods at the lower ends carries a load of 5 kN such that the cross bar remains horizontal even after the load is applied. Find the stress in each rod and the position of the load on the cross bar. Take E for steel as 200 kN/mm^2 and for copper as 100 kN/mm^2 respectively.

{ σ_{steel}= 10.61 N/mm^2; σ_{copper}= 5.305 N/mm^2; distance = 333.32 mm from copper rod end}

22. A weight of 300 kN is supported by a short concrete column having cross section as 250 mm x 250 mm. The column is strengthened in the corners by four steel bars having total cross sectional area as 5500 mm^2. Find the stresses in the steel and concrete material if E_{steel} = 15 times $E_{concrete}$. If the stress in the concrete is not to exceed 4.5 N/mm^2, what area of steel is required in order that the column could support a load of 500 kN?

{σ_{steel} = 32.25 N/mm^2, $\sigma_{concrete}$ = 2.15 N/mm^2 ; A_{steel} = 3472.2 mm^2 }

23. A bar of circular cross-section, which tapers uniformly down from a diameter of d_2 at one end to one of d_1 at the other, is subjected to an axial tensile force F. If the bar has an original length L and an elastic modulus E, derive an expression for its elongation.

If $L = 0.5m$, $d_1 = 16mm$, $d_2 = 30mm$, and $F = 48$ kN and the bar elongates an amount of 0.8mm, find the elastic modulus. Hence determine the elongation of a uniform bar of the same length and volume subjected to the same force and compare the two elongations.

{ $(4FL)/(\pi\, d_1\, d_2\, E)$; 79.578×10^3 N/mm^2; 0.704 mm; 88%, 2.066 }

24. A metal flat has a width of 10mm and length of 400mm. The depth of the flat over its length decreases from 100mm to 50mm. If the flat is subjected to a 50 kN pull at its ends, calculate the extension of the flat. Take the modulus of elasticity $E = 200$ kN/mm^2.

    {0.14 mm }

PRACTICE SHEET NO. 6 (Based on Chapter 8)

1. A vertical concrete column, 400 mm deep by 300 mm wide, carries a vertical load W which acts at 40mm and 80mm from its longitudinal and transverse axes respectively.If the allowable stresses are 10 N/mm^2 and 1 N/mm^2 in compression and tension respectively,find the maximum value of W and determine the stresses at the corners of the column.

(120 kN , 1.0 , -0.6 , -3.0 and -1.4 N/mm^2)

2. A pillar, 1.5 m high and 50 mm wide, has one vertical face. It is 50 mm deep at the top and slopes uniformly to a depth of 150 mm at the bottom. The pillar carries a vertical load of 100 kN which acts through the centroid of the top face.
Determine the value of the maximum compressive stress and the distance below the top at which it occurs.

(53.33 N/mm^2 at 0.375 m from the top)

3. A channel section, 120 mm x 80 mm x 20 mm, is used as a vertical column. It carries a vertical load (transmitted through a bracket) at a point 30 mm from its axis of symmetry and 70 mm from its back. If the maximum allowable tensile stress is 30 N/mm^2, find the values of the load and the stresses at the outer corners.

(-24.78; 23.42; -18.19 N/mm^2)

4. A concrete wall is rectangular in section and has a length of 2.0m, a thickness of 1.0m and a height of 3.0m. It is subjected to a horizontal wind pressure of 0.75 kN/m^2, which can be considered to act as a uniformly distributed load acting over the full height of the wall. If the weight of concrete is 24 kN/m^2, determine the maximum and minimum base stresses.

(92.25 kN/m^2 ; 51.75 kN/m^2)

5. Calculate the normal stresses at the four outside corners of the horizontal section of a short hollow pier of 1.5m square outside and 1.0m inside dimension, supporting a vertical point load of 140 kN on a diagonal

and located 0.8m from the centroidal vertical axis of the pier (assume the load to be placed nearer to the left hand top corner). Neglect the self-weight of the pier.
(Stress at right hand top corner = stress at bottom left hand corner = 112 kN/m^2 (comp); Stress at left hand top corner = 461.3 kN/m^2 (comp); stress at bottom right hand corner = 237.3 kN/m^2 (tensile))

6. Calculate the maximum and minimum stress intensities in a short cast iron column of hollow circular section attached with a projecting bracket carrying a load of 60 kN. The external and internal diameters of the column are 300mm and 250mm respectively. The load line is off the vertical axis of the column by 300mm.

(15.89 N/mm^2 comp; 10.34 N/mm^2 tensile)

7. Determine the limit of eccentricity which will not cause tension at any section of a column have the following cross-sectional shapes:

(a) A solid rectangular section of depth d
(b) A hollow rectangular section with outside dimensions of B and D, and internal dimensions of b and d.
(c) A solid circular section of diameter D
(d) A hollow circular section having outer and inner dimensions as D and d respectively.

{(a) $e \leq d/6$; (b) $e \leq [(BD^3 - bd^3)]/6D(BD-bd)$; (c) $e \leq d/B$; (d) $e \leq (D^2 + d^2)/BD$}

PRACTICE SHEET NO. 7 (Based on Chapter 11)

1. The shaft of a marine propeller is subjected to forces that produce compressive and shear strains of 200 and 300 microstrain respectively at a point on its surface. Calculate the principal strains and the minimum shear strain and verify the values obtained by constructing a Mohr's Circle.

(+80.28, -280.28 and +360.56 microstrain)

2. A strain gauge rosette gives three measured strains of -1400, -1020 and +570 microstrain at angles of 20^0, 140^0 and 260^0 respectively anticlockwise to the longitudinal axis of a beam. Calculate the normal and shear strains in the longitudinal and transverse directions.

If the elastic modulus is $70\text{x}10^3$ $N.mm^2$ and the shear modulus is $27\text{x}10^3$ N/mm^2, find the corresponding normal and shear stresses.

(-1807, +573 and 400 microstrain; 125.31, +2.31 and 10.8 N/mm^2)

3. At a point in a stressed material, the normal and shear stresses on a plane PQ are 95 N/mm^2 and 65 $N.mm^2$ respectively. The normal stress on the plane of maximum shear is 55 N/nn^2.

Find the principal stresses, the maximum shear stress and the angle that plane PQ makes with the major principal plane.

(131.3, -21.3 and ±76.3 N/mm^2; $29^0$12')

4. An element of a material is subjected to a two-dimensional stress system (σ_x, σ_y, τ_{yx} and τ_{xy}) such that he principal stresses are 42 N/mm^2 (tensile) and 28 N/mm^2 (compressive). The major principal stress makes an angle, ϕ, measured anti-clockwise from the x-axis such that $\phi = \tan^{-1}\frac{1}{2}$.

Calculate the values of σ_x, σ_y, τ_{xy} and τ_{yx} and illustrate the stress system on a Mohr's Circle and a sketch of the element.

(28, -14 and 28 N/mm^2)

5. An element in a material in a material is subjected to bi-axial stresses of 100 N/mm^2 in tension and 70 N/mm^2 in compression. If the major principal stress is 125 N/mm^2, use a Mohr's Circle to determine the values of the minor principal and maximum shear stresses. Hence, calculate the shear stresses on the sides of the element and the angle which the major principal stress makes with the direction of the tensile stress of 100 N/mm^2.

(-95 and ± 110 N/mm^2; 69.82 N/mm^2 and 19^0 42')

6. The readings from a 45^0 strain-gauge rosette taken in order are 400, 200 and -100 microstrain. If the elastic modulus and Poisson's ratio of the material are 210×10^3 N/mm^2 and 0.30 respectively, find the principal and maximum shear stresses and their directions. Find also the stresses in the directions of the two end gauges and the value of the modulus of rigidity.

(405, -105
(86.18, 3.82 and ±41.18 N/mm^2 $5^0$18' and $95^0$18')
(85.4, 4.6 N/mm^2 and 80.8 N/mm^2)

7. Explain the meaning of the terms Modulus of Elasticity and Poisson's ratio and, hence, find expressions for the two-dimensional stress and strain in terms of each other. Find also an expression for the increase in area of a rectangular plate subjected to two-dimensional strain. Illustrate your answers fully with neat sketches.

A rectangular plate, 600 mm long by 400 mm wide, is subjected to a tensile stress of 165 N/mm^2 in the long direction and thereby changes dimensions by 0.48 and 0.104 mm respectively in the longitudinal and transverse directions. Find the Modulus of Elasticity and Poisson's ratio. If the plate is also subjected to a tensile stress of 110 N/mm^2 in the transverse direction, find the percentage increase in area.

{ $A(\varepsilon_x + \varepsilon_y)$; 206.25×10^3 N/mm^2; 0.325; 0.09%}

8. In an element of material, subjected to general two-dimensional stress one axial stress is 66 N/mm^2 (tensile) and the shear stress is 48 N.mm^2. Calculate the values and directions of the principal stresses and the normal and shear stresses on planes equally inclined to the axes if the other axial stress of 22 N/mm^2 is (a) tensile and (b) compressive.

Show the axial, shear and principal stresses on neat sketches of the element and show the entire stress system on Mohr's Stress Circles drawn to scale.
{96.8 and -8.8 N/mm^2; 32.69^0; 92 and 22, -4 and -22 N/mm^2 }
{87.16 and -43.115 N/mm^2; 23.745^0; 70, 44, -26 and -44 N/mm^2}

9. If the normal to a plane pq makes an angle of Q with the x-axis in a general two-dimensional x-y stress system, write down the equations for the normal and shear stresses on the plane. Hence using the fact that there are no shear stresses on the principal planes, write down expressions for the normal and shear stresses on plane pq in terms of the principal stresses and the angle (a) between the normal stress and the major principal stress.

The normal and shear stresses on such a plane pq are 80 and 21 N / mm^2 respectively and the normal stress on a plane at an angle of 60^0 to plane pq is 50 N/mm^2. Calculate the principal stresses, the maximum shear stress, the angle of 60^0 to the plane pq. Hence, draw Mohr's Stress Circle to scale, illustrating clearly all the important values of stress. Deduce, from the circle, the values of the normal stresses on the planes at right angles to the plane pq and the one at an angle of 60^0 to plane pq.

(94.55, 49.7 and ±22.43 N/mm^2)

10. An element of a material is subjected to a two-dimensional state of stress (σ_x, σ_y, τ_{xy}, τ_{yx}) and a plane pq through the element is so located that its normal makes an angle θ with the direction of σ_x. If $\sigma_x > \sigma_y$, derive expressions for the normal and shear stresses on the plane and illustrate the state of stress on a Mohr's Circle of Stress. Verify the Circle and

deduce from it expressions for the principal stresses and their directions. Find also an expression for the maximum shear stress.

11. A strain rosette gives readings of 460, 200 and -165 microstrain at angles of 60^0 apart. Calculate the magnitude, type and direction of the principal strains and illustrate the state of strain clearly on a Mohr's Circle drawn to scale.

If the modulus of elasticity and Poisson's ratio are 207×10^3 N/mm^2 and 0.29 respectively, calculate the principal stresses and illustrate these stresses on a sketch of the element.

(527.54 and -197.54 microstrains, $17{,}77^0$; 106.28 and -10.07 N/mm^2)

12.A 60^0 strain rosette on an element of stressed material measures 600, -200 and 200 microstrain in that order. Find the magnitude and directions of the principal strains and illustrate the whole strain system clearly on a properly dimensioned Mohr's Circle and a sketch of the element. For an elastic modulus of 207×10^3 N/mm^2 and Poisson's ratio of 0.28, find the principal stresses.

(666.88, -261.88, 132.45 and -17.195 N/mm^2)

13. An element in a structure subjected to a state of general two-dimensional stress is found to have a major principal tensile stress of 160 N/mm^2 and a maximum shear stress of 120 N/mm^2. Calculate the minor principal stress.

If the major principal stress makes an angle of 30^0 with the larger axial stress, calculate the axial and shear stresses on the element and show the complete stress system clearly on a sketch of the element and a properly dimensioned Mohr's circle.

Determine also the axial stains if the Elastic Modulus is 207×10^3 N/mm^2 and Poisson's ratio is 0.30.

(-80; 100, -20 and 103.92 N/mm^2; 512.077 N/mm^2 and -241.546)

14. An element in a two-dimensional xy stress system is subjected to a tensile stress, σ_x, of 64 N/mm^2 and a compressive stress, σ_y, of 48 N/mm^2. If the major principal stress is limited to 80 N/mm^2, calculate the shear stresses τ_{xy} and τ_{yx}, the maximum shear stress and the minor principle stress. Calculate also the directions of the principal and shear stresses.

Illustrate the calculated answers on a scaled diagram of Mohr's circle and show the position and direction of the principal and maximum shear stresses on a sketch of the element.

{45.255, 72 and -64 N/mm^2; 19.471^0 and 64.471^0}

15.Explain the meaning of the term Hoop Stress and list the most common conditions which give rise to it. Explain also the meaning of the term Poisson's Ratio. Illustrate your answer with the help of sketches.

16. At a point on the surface of a material, readings from 60^0-strain gauge rosette are 520, 280 and -200 microstrain. Find the magnitude and direction of the principal strains and the value of the maximum shear strain. Illustrate them on both a strain diagram and a properly dimensioned diagram of Mohr's circle.

If the moduli of Elasticity and Rigidity are 208x10^3 and 80x10^3 N/mm^2 respectively and Poisson's ratio is 0.29, deduce the values of the principal and maximum shear stresses.

{623.32, -223.32 and ±846.64 microstrains; 126.85, -9.665 and ±67.73 N/mm^2}

17. Explain the meaning of the terms principal planes, principal stresses and planes of maximum shear stress.

On an element in a material, the biaxial stresses are 100 N/mm^2 (tensile) and 60 N/mm^2 (compressive) respectively and the major principal stress is 120 N/mm^2. Calculate the coordinates of the centre of Mohr's stress circle and, hence, the minor principal stress, maximum shear stresses and the shear stresses on the faces of the element. Determine also the directions of the principal stresses and illustrate the entire stress system on a sketch of the element and a diagram (drawn to scale) of Mohr's stress circle.

{20,0; -80, ±100 and ±60 N/mm^2; 18.435^0}

18. At a stressed point in a material, the major principal stress is 120 N/mm^2 (tensile) and the maximum shear stress is 109 N/mm^2. Find the minor principal stress.

If the major principal stress acts at an angle of $\tan^{-1}(\frac{1}{2})$ to the x-direction of a general two-dimensional x-y system of stresses, find these stresses. Illustrate the complete stress system on a sketch of the element and a Mohr's stress circle.

{-98, 76.4, -54.4 and ±87.2 N/mm^2}

PRACTICE SHEET NO. 8 (Based on Chapter 9)

1. A uniform horizontal cantilever has a span l, an elastic modulus E and a second moment of area I. If a concentrated vertical load W acts at a point three quarters from the fixed end, show that the slope at that point is $9Wl^2/32EI$. Show also that the deflection at the free end is one and a half times that at the load point.

2.A uniform horizontal beam is simply supported at A and B and carries a load varying uniformly from zero at A to 2w at B. If the beam has a span of l, derive its deflection equation and determine the position and magnitude of the maximum deflection.

{0.52 l from A, $0.013\ wl^4 / EI$}

3. A uniform horizontal cantilever of span l (flexure rigidity EI) carries a vertical load varying uniformly from zero at a point quarter of the span from the free end to w at the fixed end. Show that the deflection at the free end is approximately equal to $0.015\ wl^4/EI$. If this end is now supported by a vertical prop such that both ends are at the same level show that the prop carries a force of approximately 0.045 wl.

4. A uniform horizontal cantilever, 2.5 m long, carries a vertical concentrated load of W at a distance of 1.5 m from the fixed end. If the beam has a flexure rigidity of EI, determine the deflection at the free end. If a similar load is now placed at the free end such that both loads together cause a deflection at that end equal to one caused by a single vertical load of 50 kN there, find the value of W. Calculate the ratio of the maximum stress in the two-load condition to that in the single 50 kN - load condition.

{2.25 W/EI m; 34.9 kN; 1.117}

5. A horizontal cantilever with a span of 3 m, a second moment of area of $800 \times 10^4\ mm^4$ and an elastic modulus of $200 \times 10^3\ N/mm^2$ carries a

vertical concentrated load of 60 kN at a point 2 m from the fixed end. If the free end is attached to a hanging vertical tie-rod with a length of 4 m and a cross-sectional area of 400 mm^2, what is the force in the tie-rod and the actual deflection at that end.

{30.837 kN and 1.542 mm}

6. A horizontal beam, which has a second moment of area of 1600×10^5 mm^4 and an elastic modulus of 200×10^3 N/mm^2, is simply supported over a span of 8 m and carries a uniformly distributed load of 24 kN/m. If the beam is supported at the centre such that the deflection there is only 10 mm, calculate the reactions at the three supports and determine the principal values of shearing force and bending moment.

{51,51 and 90 kN at ends and centre; 51 and 45 kN at ends and centre; M_B = 12 kN m and M_{max} = 54.1875 kN m at 2.125 from end supports}

7. A uniform horizontal cantilever ABC has its fixed and free ends at A and C respectively and carries a concentrated vertical load at a point B between the ends. Derive the slope and deflexion equations and find an expression for the deflexion at the free end.

Such a cantilever carries a uniformly distributed load of 30 kN/m over a span of 3.2 m and a concentrated vertical load at a distance of 2.4 m from the fixed end. The cantilever has a depth of 600 mm, Second Moment of area of 68400×10^4 mm^4 and Modulus of Elasticity of 210×10^3 N/mm^2. If the allowable bending stress is 40 N/mm^2 and the maximum deflection must not exceed 6.3 mm, find the value of the concentrated load.

($\{-wa^2[3l - a]\}/\{6EI\}$; 69 kN)

8. Derive the deflexion equation and find the maximum deflexion for a uniform horizontal beam carrying a uniformly distributed load throughout its span.

A horizontal beam, 4 m long and carrying a uniformly distributed load of 30 kN/m, is kept in position by three vertical rods suspended from a rigid ceiling. Two of the rods (diameter = 25 mm) are at each end of the beam and the third rod (diameter = 30 mm) is at the centre, the length of each rod being 2.4 m. If the second moment of area of the beam is 8 800x10^4 mm^4 and the Modulus of Elasticity of the beam and rods is 210x10^3 N/mm^2, calculate the stresses in the rods and the maximum deflection of the beam.

{96.355 N/mm^2 (centre) and 52.856 N/mm^2 (end); 0.497 mm}

9. A uniform horizontal beam, 6 m long and simply supported at its ends, carries a uniformly distributed load of 36 kN/m between points 1 m and 3 m from the left hand end. Find the position of the maximum deflection. If the elastic modulus and second moment of area are 200x10^3 N/mm^2 and 2x10^8 mm^4 respectively, determine the value of this deflection and those at intervals of 1 m along the beam. Hence, draw the deflection diagram to scale on graph paper, showing clearly thevalues determined.

{2.785 m from A; 6.6435 mm; 3.6, 6.0375, 6.6, 5.4 and 3 mm}

PRACTICE SHEET NO. 9 (Based on Chapter 10)

1. A solid cylindrical shaft, 2.1 m long, has a diameter of 200 mm for two-thirds of its length and a diameter of 100 mm for the remainder. If the total angle of twist is limited to 2^0 and the modulus of rigidity = 80×10^3 N/mm^2, find the power transmitted by the shaft at 500 r.p.m.

(1822.78 kN)

2. A hollow cylindrical shaft has outer and inner diameters of 400 mm and 150 mm respectively. The shaft, which is horizontal, has a length of 3.2 m and it is rigidly fixed at one end. It is subjected to an axial torque of 30 kN m, an axial thrust of 40 kN and a downward vertical load of 20 kN at the free end. Find the principal stresses and maximum shear stress at a point on the upper surface of the shaft three-quarters of the distance from the fixed end.

(3.79, -1.56 and 2.68 N/mm^2)

3. A solid shaft, 0.75 m long and 50 mm in diameter, has a concentric hole drilled for a portion of its length and is subjected to an axial torque of 1.67 kN m. The modulus of rigidity is 30×10^3 N/mm^2. If the shearing stress and angle of twist are not to exceed 75 N/mm^2 and $1\frac{1}{2}^0$ respectively, find the maximum length and diameter of the hole.

(0.191 m; 27.6 mm)

4. A cylindrical steel rod is enclosed by a close-fitting duralumin tube, the two being securely fastened to form a composite shaft. The moduli of rigidity of steel and duralumin are 78 N/mm^2 and 26 N/mm^2 respectively. If the shaft is subjected to an axial torque of 0.7 kN m and the shearing stresses int the steel and duralumin are not to exceed 90 N/mm^2 respectively, find the diameters of the rod and tube.Find also the angle of twist on a length of 1.2m.

(18.76 mm and 37.52 mm; 8.46^0)

5. A solid cylindrical shaft with a diameter of 127 mm is to be replaced by a hollow one of the same material and strength. If the ratio of the outer to inner

diameters of the hollow shaft is 1.5, find these diameters and the percentage saving in material.

(136.67 mm and 91.11 mm; 35.7 %)

6. A hollow shaft, having an external diameter of 400 mm and a modulus of rigidity of 80×10^3 N/mm^2 transmits 10 MW at 150 revolutions per minute. If the angle of twist of the shaft over its length of 2.4 m is 0.52^0, find the internal diameter of the shaft and the maximum shear stress.

(254.043 mm and 60.504 N/mm^2)

7. A composite shaft consists of a solid steel rod of 75 mm diameter surrounded by a closely fitting brass tube of the same length firmly fixed to it. If the rod and tube share equally an applied torque and the steel has a modulus of rigidity equal to twice that of the brass, find the outer diameter of the tube. For a shaft of length 4 m, applied torque of 16.2 kN m and modulus of rigidity of steel of 80×10^3 N/mm^2, calculate the maximum stress in each material and the angle of twist.

(98.7 mm;97.78 and 64.36 N/mm^2 for steel and brass; 7.47^0)

8. Derive an expression for the axial torque on hollow and solid cylindrical shafts in terms of shear stress and radius.

A solid cylindrical shaft transmits power of 420 kW at an angular velocity of 4p radian/second. If the allowable shear stress is 93 N/mm^2, find the diameter of the shaft. Find also the diameters of a replacement hollow shaft of the same length and material (the internal diameter being two thirds of the external diameter) and the percentage saving in weight. Find the minimum shear stress in the hollow shaft and sketch the stress distribution in both shafts.

For a length of 3.8 m and Modulus of rigidity of 78×10^3 N/mm^2 what is the angle of twist in the hollow shaft?

(122.323 mm; 87.6 and 131.633 mm, 35.67 %; 62 N/mm^2, 3.944^0)